

## 7.1A DIRECTIVITY AND SPACING FOR THE ANTENNA ELEMENTS

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While deciding on the optimum design choice for the MST radar antenna, the following factors are required to be considered: directivity and gain; beam width and its symmetry; sidelobe levels - near and wide angle; impedance matching; feeder network losses; polarisation diversity; steerability; cost-effectiveness; and maintainability.

The scope of this note will be restricted to the directivity and related beam-forming aspects of various antenna elements and also the directivity aspects when such elements are formed into an array. Array performance will be considered in regard to important variables, in particular, the spacing of the elements.

## ANTENNA CONFIGURATIONS

Alternative configurations possible for MST radar antenna are the following: coaxial collinear array; discrete dipole array; Yagi array; dish antenna; and short backfire array.

## COAXIAL COLLINEAR ARRAY

A collinear antenna (BALSLEY and ECKLUND, 1972) is constructed of a series of half wavelengths of coaxial cable that have been connected together by electrically interchanging the inner and outer conductors at each junction. A half-wave dipole has a length higher than  $0.5\lambda$  to compensate for the propagation velocity of the cable ( $0.67\lambda$  for RG-8 cable). The number of dipoles in a collinear antenna could be any even number and typical cases are 26-element, 16-element etc.

The collinear antenna (BALSLEY and ECKLUND, 1972), by virtue of its construction of large numbers of radiators, inherently has high directivity in the plane of the antenna. A 26-element collinear antenna has shown a theoretical beam width of  $5.6^\circ$  at 49.8 MHz. The directivity in the plane perpendicular to the collinear antenna is isotropic, as applicable in the case of any dipole.

The radiation pattern of a collinear antenna of  $n$  halfwave dipoles fed at the center symmetrically (BALSLEY and ECKLUND, 1972) is expressed as

$$E_T = \frac{2 \cos\left(\frac{\pi}{2} \sin\theta\right)}{\cos\theta} \sum_{K=0}^{K=\frac{n-2}{2}} A_K \cos\left(\frac{2K+1}{2} \psi\right)$$

Where  $\theta$  is the angle from the broadside axis,  $A_K$  the amplitude of  $K^{\text{th}}$  element from the center and  $\psi = \left(\frac{2\pi}{\lambda}\right)d \sin\theta$ ,  $d$  being distance between elements.

The collinear antenna when built into an array, as done in the case of Sunset (GREEN et al., 1981), Poker Flat radars (BALSLEY et al., 1981) of  $M \times N$  elements produces a directive pattern on the broadside of the array. The array pattern is obtained from a product of the element pattern and the array space factor. The array space factor (ALLMAN and BOWHILL, 1976) is given by

$$S_{xy} = S_x S_y = \frac{1}{M} \left| \frac{\sin(M\psi_x/2)}{\sin(\psi_x/2)} \right| \cdot \frac{1}{N} \left| \frac{\sin(N\psi_y/2)}{\sin(\psi_y/2)} \right|$$

The spacing elements in the plane of the collinear antenna does not provide much scope for adjustment. In the plane perpendicular to the antenna elements, the spacing should be decided from consideration of the following factors:

- (1) A close spacing, say  $0.5\lambda$  or less, would result in strong mutual coupling leading to large impedance variations. Also, the array area for a given number of elements and hence the directivity, would be reduced.
- (2) A large spacing, say  $1.0\lambda$  or higher causes generation of grating lobes and limits the angle of steerability.

A spacing between  $0.5\lambda$  to  $1.0\lambda$  is considered acceptable in practical situations and  $0.6\lambda$  to  $0.8\lambda$  may be deemed as optimum (KRAUS, 1950).

Further, spacing of the dipole from the ground influences the directivity and impedance. For a half-wave dipole, highest directivity (KRAUS, 1950) is realised for spacings between  $0.1\lambda$  and  $0.2\lambda$ , although the bandwidth will be narrow and the input impedance will be sensitive to ground variations, for such spacings. A spacing of  $\lambda/4$  is considered optimum from considerations of good bandwidth, tolerance to ground undulations and minimum reflector losses. When wide-angle steerability is required, a spacing of  $3\lambda/8$  can ensure better gain performance at wide angles (OLINER and MALECH, 1966).

Directivity of an array of linear dipoles is also affected to some extent by mutual coupling. However, when the array is very large and dipoles are backed by a reflector, the effect of mutual coupling is negligible and the gain varies only by 0.1 dB from the value computed from the physical area of the array (DEVANE and DION, 1962).

The directivity of the array also depends on the amplitude illumination of the elements. While a uniform illumination gives maximum directivity of unity but poor sidelobe level (theoretically 13.2 dB), a tapered illumination (KOSHY et al., 1983) can improve the sidelobe level by 6 to 8 dB, at the expense of reduction in directivity of about 1 to 1.5 dB.

#### DISCRETE DIPOLE ARRAY

Dipoles, half-wave or full-wave, fabricated out of aluminum tubes form the basic element. The beam is isotropic in the plane perpendicular to the dipole and moderately directive in the plane of the dipole. The 3-dB beam width for half-wave and full-wave dipoles are  $78^\circ$  and  $47^\circ$ , respectively.

An array of dipoles is constructed by arranging them collinear in columns and with several rows of such columns. Directivity of an array of dipoles is dependent on the number of elements in both planes, element directivity, spacing etc., the same way as in the case of the coaxial collinear antenna explained earlier. Excellent design details are provided by ALLMAN and BOWHILL (1976), BOWHILL and MAYES (1979), and MAYES and TANNER (1981) as part of their work on the Urbana MST radar.

Both the coaxial collinear and discrete dipole arrays require two independent orthogonal arrays to produce polarization in E-W and N-S directions. In respect of steering, while collinear coaxial array is capable of steering only in the plane perpendicular to the dipoles, the discrete dipole array can be made to steer in orthogonal planes by providing suitable progressive phase shifts in both planes (MAYES and TANNER, 1981).

### YAGI ARRAY

Yagi antenna element is compact in hardware and provides a high directivity in both principal planes. A four-element Yagi antenna, as used in the SOUSY (CZECHOWSKY and MAYER, 1980) and MUR (FUKAO et al., 1980) gives a gain of 8.7 dB.

When formed into an array, the Yagi type requires a lesser number of elements in comparison to the dipole type, for realising a given directivity. The array directivity depends on the number of elements and is computed by the standard pattern multiplication method. The Yagi array, owing to the directive element pattern and discrete-element aperture available to illumination taper, can ensure low sidelobes (FUKAO et al., 1980).

The effect of spacing of elements on directivity of a Yagi array is governed by similar considerations as in dipole arrays. At close distances, where mutual coupling would be significant, the element factor generally sharpens so as to improve the directivity in low-elevation angles (FUKAO et al., 1980).

### DISH ANTENNA

The dish antenna is a single element antenna consisting of a large parabolic reflector and a prime-focus or cassegranian feed system.

The directivity of the dish antenna depends primarily on the aperture area of the reflector (SILVER, 1949) and the gain is expressed by

$$g = K \left( \frac{4\pi A}{\lambda^2} \right)$$

where K is the antenna efficiency which is the product of a number of efficiency factors considering losses due to aperture illumination, spillover, blockage, surface error, VSWR, cross polarization, etc. By proper control of the feed illumination, very low sidelobe levels can be achieved in dish antennas with moderate loss of gain.

The only 'spacing' involved in dish antennas is that of the feed which is required to be at a unique position, namely, the focus of the reflector for optimum directivity. Movement of the feed in the focal plane perpendicular to the axis of the dish gives rise to steering of the beam.

### SHORT BACKFIRE ANTENNA

The short backfire antenna (EHRENSPECK, 1965; EHRENSPECK and STROM, 1977) is a compact high-gain element, consisting of a slotted dipole feed, a reflector disc of diameter  $0.5\lambda$  and a circular planar reflector with a rim of width  $0.5\lambda$ . For a planar reflector diameter of  $2\lambda$ , this antenna gives a gain of 15.1 dB.

In an array, the high element gain of the SBF antenna gives the advantage of high array directivity for a small number of elements. A  $16 \times 16$  uniform array of 2 circular SBF antennas can realise a gain of 39 dB (PHYSICAL RESEARCH LABORATORY, 1981).

Nevertheless, the inter-element spacing of  $2\lambda$  required in such an array would produce grating lobes at  $30^\circ$  and  $90^\circ$  from the array axis and hence would result in high sidelobe levels at these angles. Good sidelobe levels, better than 20 dB, can be achieved by reducing the element spacing to  $1\lambda$  (EHRENSPECK and STROM, 1977). Consequent to this, the gain of the  $16 \times 16$  element array

would, however, reduce to around 34 dB.

#### CONCLUSION

In the foregoing sections, a brief recapitulation is given of the directivity and related features of the various elements available for realizing an optimum antenna for MST radar. The actual choice of a particular element and array configuration should, however, be based not only on directivity but also on a number of other considerations as mentioned in the first paragraph of this paper.

#### REFERENCES

Allman, M. E. and S. A. Bowhill (1976), Feed system design for the Urbana incoherent scatter radar antenna, Aeron. Rep. No. 71, Aeron. Lab., Dep. Elec. Eng., Univ. Ill., Urbana-Champaign.

Balsley, B. B. and W. L. Ecklund (1972), A portable coaxial collinear antenna, IEE Trans. on Antennas and Prop., AP-20, 513-516.

Balsley, B. B., W. L. Ecklund, D. A. Carter, P. E. Johnston and A. C. Riddle (1981), The Poker Flat MST radar system: Current status and capabilities, 20th Conference on Radar Meteorology, Nov. 30 - Dec. 3.

Bowhill, S. A. and P. E. Mayes (1979), Proposal for engineering and cost study of a phased array antenna for Urbana coherent scatter radar, Aeron. Lab., Dep. Elec. Eng., Univ. Ill., Urbana-Champaign.

Czechowsky, P. and K. Mayer (1980), Antenna system for SOUSY VHF radars, Max-Planck Institut fur Aeronomie Report MPAE - T-00-80-19.

Devane, M. E. and A. R. Dion (1962), The El Campo solar radar antenna, MIT Lincoln Lab., Tech. Rep. No. 276.

Ehrenspeck, H. W. (1965), The short backfire antenna, Proc. IEEE, 53, 1138-1140.

Ehrenspeck, H. W. and J. A. Strom (1977), Short backfire antenna - a highly efficient array element, Microwave Journal, 20, 47-49.

Fukao, S., S. Kato, T. Aso, M. Sasada and T. Makihira (1980), Middle and upper atmosphere radar (MUR) under design in Japan, Radio Sci., 15, 225-231.

Green, J. L., J. M. Warnoch, W. L. Clark, F. J. Eggert and K. J. Ruth (1981), Modification to the sunset radar to provide antenna beam steering, 20th Conference on Radar Meteorology, Nov. 30 - Dec. 3.

Koshy, V. K., A. K. Majumdar and K. K. Gupta (1983), Non-uniform excitation of a coaxial collinear array (to be published).

Kraus, J. D. (1950), Antennas, Mc-Graw Hill, New York.

Mayes, P. E. and D. R. Tanner (1981), Preliminary draft on design studies of phased array for Urbana coherent scatter radar, Aeron. Lab., Dep. Elec. Eng., Univ. Ill., Urbana-Champaign.

Oliner, A. A. and R. G. Malech (1966), Microwave Scanning Antennas, (ed. R. C. Hansen), Academic Press, New York.

Physical Research Laboratory (1981), A proposal for atmospheric studies using high power VHF radar, Physical Research Laboratory, Ahmedabad, India.

Silver, S. (1949), Microwave Antenna - Theory and Design, Mc-Graw Hill, New York.

## APPENDIX

### COAXIAL COLLINEAR ANTENNA ARRAY

#### ELEMENT PATTERN

$$E_T = \frac{2 \cos(\frac{\pi}{2} \sin\theta)}{\cos\theta} \sum_{k=0}^{n-2} A_k \cos(\frac{2k+1}{2} \psi)$$

$$\text{where } \psi = \frac{2\pi}{\pi} d \sin\theta$$

#### ARRAY FACTOR

$$S_{xy} = \frac{1}{M} \left| \frac{\sin(M\psi_x/2)}{\sin(\psi_x/2)} \right| \frac{1}{N} \left| \frac{\sin(N\psi_y/2)}{\sin(\psi_y/2)} \right|$$

#### MAXIMUM GAIN OR DIRECTIVITY OF ARRAY

$$G = \frac{4\pi |E_T(0,0) S_{xy}(0,0)|^2}{\int_0^{2\pi} \int_0^{\pi/2} |E_T(\theta, \phi) S_{xy}(\theta, \phi)|^2 \sin\theta d\theta d\phi}$$

Where there is no radiation below the ground plane.

### DISCRETE DIPOLE ANTENNA ARRAY

#### ELEMENT PATTERN

$$E_T = \frac{2 \cos(\frac{\pi}{2} \sin\theta)}{\cos\theta}$$

#### ARRAY FACTOR

$$S_{xy} = \frac{1}{M} \left| \frac{\sin(M\psi_x/2)}{\sin(\psi_x/2)} \right| \frac{1}{N} \left| \frac{\sin(N\psi_y/2)}{\sin(\psi_y/2)} \right|$$

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where there is no radiation below the ground plane.

YAGI ARRAY

## ELEMENT PATTERN

$$E(\theta) = \frac{\sin(kL/2)}{(kL/2)} \frac{(\cos\theta - K)}{(\cos\theta - K)}$$

$$K = \frac{k_x}{k} = \frac{0.468 + L/\lambda}{L/\lambda}$$

Where  $L$  = length of the Yagi Element

$$\cos k_x s = \cos ks - 1/2 e^{ksf(h,a)}$$

$$f(h,a) = \frac{\cos kh(\frac{\sin kh}{kh} - kh \cos kh(\frac{\cos ka}{ka}))}{\sin kh - kh \cos kh}$$

$$\text{for } s \gg a, h \gg a, s \gg \frac{a}{2}$$

## ARRAY FACTOR

$$S_{xy} = \frac{1}{M} \left| \frac{\sin(M\psi_x/2)}{\sin(\psi_x/2)} \right| \frac{1}{N} \left| \frac{\sin(N\psi_y/2)}{\sin(\psi_y/2)} \right|$$

## MAXIMUM GAIN OR DIRECTIVITY OF ARRAY

$$G = \frac{4\pi |E_T(0,0) S_{xy}(0,0)|^2}{\int_0^{2\pi} \int_0^{\pi/2} |E_T(\theta, \phi) S_{xy}(\theta, \phi)|^2 \sin \theta d\theta d\phi}$$

Where there is no radiation below the ground plane

DISH ANTENNA

## FIELD PATTERN

E plane:

$$E_\theta \Big|_{\phi=\pi/2} = \frac{j\omega\mu_0}{2\pi R} e^{-jk_o(R+2f)} \left( \frac{1}{\zeta_o} \frac{P_t}{2\pi} \right)^{1/2} \cos \theta$$

$$\int_0^a \int_0^{2\pi} \frac{[G_f(\theta', \phi')]^{1/2}}{P} e^{jk_o r \sin \theta \sin \phi' r d\phi'} dr$$

H plane:

$$E_\phi \Big|_{\phi=0} = \frac{j\omega\mu_0}{2\pi R} e^{-jk_o(R+2f)} \left( \frac{1}{\zeta_o} \frac{P_t}{2\pi} \right)^{1/2}$$

$$\int_0^a \int_0^{2\pi} \frac{[G_f(\theta', \phi')]^{1/2}}{P} e^{jk_o r \sin \theta \cos \phi' r d\phi'} dr$$

$\zeta$  = free space impedance.

$$\theta' = 2 \tan - \frac{1r}{2f}$$

$$P = \frac{4f^2 + r^2}{4f}$$

$$e_{ry} = \frac{(\sin^2 \phi' \cos \theta' + \cos^2 \phi')}{\sqrt{1 - \sin^2 \phi' \sin^2 \theta'}}$$

$$\text{Gain} = k \left( \frac{4\pi A}{\lambda^2} \right)$$

Where 'k' is efficiency.

#### SHORT BACKFIRE ANTENNA ARRAY

##### ELEMENT PATTERN

E-plane

$$E_\theta = I_E(A, \theta) \sin \phi$$

H-plane

$$E_\phi = I_H(A, \theta') \cos \phi$$

$$I_E(A, \theta) = (1 + \cos \theta) \int_0^A \frac{\sqrt{A^2 - z^2}}{1 - \{4 \sin \theta \sqrt{(A^2 - z^2)}\}^2} \cos \frac{\pi z}{2A} \cos(2\pi \sin \theta \sqrt{(A^2 - z^2)} dz$$

$$I_H(A, \theta') = (1 + \sin \theta') \int_0^A \sqrt{A^2 - z^2} \cos \frac{\pi z}{2A} \cos(2\pi z \cos \theta') dz$$

where  $\theta' = \theta - 90^\circ$

$$A = \frac{D_M}{2} \text{ is the electrical aperture radius} > \text{physical aperture radius}$$

##### ARRAY FACTOR

$$S_{xy} = \frac{1}{M} \left| \frac{\sin(M\psi_x/2)}{\sin(\psi_x/2)} \right| \frac{1}{N} \left| \frac{\sin(N\psi_y/2)}{\sin(\psi_y/2)} \right|$$

##### MAXIMUM GAIN OR DIRECTIVITY OF ARRAY

$$G = \frac{4\pi |E_T(0,0) S_{xy}(0,0)|^2}{\int_0^{2\pi} \int_0^{\pi/2} |E_T(\theta, \phi) S_{xy}(\theta, \phi)|^2 \sin \theta d\theta d\phi}$$

Where there is no radiation below the ground plane.